

Probabilistic and Causal Inference

The Works of Judea Pearl

Hector Geffner
Rina Dechter
Joseph Y. Halpern
(Editors)



ASSOCIATION FOR COMPUTING MACHINERY

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