Probabilistic and Causal Inference
The Works of Judea Pearl
Hector Geffner, Rina Dechter, Joseph Y. Halpern (Editors)

Professor Judea Pearl won the 2011 Turing Award "for fundamental contributions to artificial intelligence through the development of a calculus for probabilistic and causal reasoning." This book contains the original articles that led to the award, as well as other seminal works, divided into four parts: heuristic search, probabilistic reasoning, causality, first period (1988–2001), and causality, recent period (2002–2020). Each of these parts starts with an introduction written by Judea Pearl. The volume also contains original, contributed articles by leading researchers that analyze, extend, or assess the influence of Pearl’s work in different fields: from AI, Machine Learning, and Statistics to Cognitive Science, Philosophy, and the Social Sciences. The first part of the volume includes a biography, a transcript of his Turing Award Lecture, two interviews, and a selected bibliography annotated by him.
Contents

Preface xxv

Credits xxvii

PART I  INTRODUCTION  1

Chapter 1  Biography of Judea Pearl by Stuart J. Russell  3

References  9

Chapter 2  Turing Award Lecture  11

References  27

Chapter 3  Interview by Martin Ford  29

References  42

Chapter 4  An Interview with Ron Wassertein on How The Book of Why Transforms Statistics  43

Chapter 5  Selected Annotated Bibliography by Judea Pearl  49

Search and Heuristics  49
Bayesian Networks  50
Causality  51
Causal, Casual, and Curious  53

PART II  HEURISTICS  57

Chapter 6  Introduction by Judea Pearl  59

References  60
Chapter 7  Asymptotic Properties of Minimax Trees and Game-Searching Procedures  61
Judea Pearl
Abstract  61
7.1 The Probability of Winning a Standard \( h \)-level Game Tree with Random WIN Positions  62
7.2 Game Trees with an Arbitrary Distribution of Terminal Values  65
7.3 The Mean Complexity of Solving \((h, d, P_0)\)-game  69
7.4 Solving, Testing, and Evaluating Game Trees  75
7.5 Test and, if Necessary, Evaluate—The SCOUT Algorithm  78
7.6 Analysis of SCOUT’s Expected Performance  79
7.7 On the Branching Factor of the ALPHA–BETA \((\alpha–\beta)\) procedure  85
References  88

Chapter 8  The Solution for the Branching Factor of the Alpha–Beta Pruning Algorithm and its Optimality  91
Judea Pearl
8.1 Introduction  92
8.2 Analysis  94
8.3 Conclusions  101
References  102

Chapter 9  On the Discovery and Generation of Certain Heuristics  103
Judea Pearl
Abstract  103
9.1 Introduction: Typical Uses of Heuristics  103
9.2 Mechanical Generation of Admissible Heuristics  114
9.3 Can a Program Tell an Easy Problem When It Sees One?  117
9.4 Conclusions  119
References  121

PART III  PROBABILITIES  123

Chapter 10  Introduction by Judea Pearl  125
References  126

Chapter 11  Reverend Bayes on Inference Engines: A Distributed Hierarchical Approach  129
Judea Pearl
Abstract  129
Chapter 12  Fusion, Propagation, and Structuring in Belief Networks  139
   Judea Pearl
   Abstract  139
12.1  Introduction  140
12.2  Fusion and Propagation  148
12.3  Structuring Causal Trees  169
12.A  Appendix A. Derivation of the Updating Rules for Singly Connected Networks  181
12.B  Appendix B. Conditions for Star-decomposability  183
   Acknowledgments  185
   References  186

Chapter 13  GRAPHOIDS: Graph-Based Logic for Reasoning about Relevance Relations Or When Would x Tell You More about y If You Already Know z?  189
   Judea Pearl and Azaria Paz
   Abstract  189
13.1  Introduction  190
13.2  Probabilistic Dependencies and their Graphical Representation  192
13.3  GRAPHOIDS  195
13.4  Special Graphoids and Open Problems  196
13.5  Conclusions  198
   References  199

Chapter 14  System Z: A Natural Ordering of Defaults with Tractable Applications to Nonmonotonic Reasoning  201
   Judea Pearl
   Abstract  201
<table>
<thead>
<tr>
<th>PART IV</th>
<th>CAUSALITY 1988–2001</th>
<th>215</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 15</td>
<td>Introduction by Judea Pearl</td>
<td>217</td>
</tr>
<tr>
<td>References</td>
<td>219</td>
<td></td>
</tr>
<tr>
<td>Chapter 16</td>
<td>Equivalence and Synthesis of Causal Models</td>
<td>221</td>
</tr>
<tr>
<td>TS Verma and Judea Pearl</td>
<td>221</td>
<td></td>
</tr>
<tr>
<td>Abstract</td>
<td>221</td>
<td></td>
</tr>
<tr>
<td>16.1</td>
<td>Introduction</td>
<td>222</td>
</tr>
<tr>
<td>16.2</td>
<td>Patterns of Causal Models</td>
<td>224</td>
</tr>
<tr>
<td>16.3</td>
<td>Embedded Causal Models</td>
<td>227</td>
</tr>
<tr>
<td>16.4</td>
<td>Applications to the Synthesis of Causal Models</td>
<td>231</td>
</tr>
<tr>
<td>IC-Algorithm (Inductive Causation)</td>
<td>232</td>
<td></td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>234</td>
<td></td>
</tr>
<tr>
<td>References</td>
<td>234</td>
<td></td>
</tr>
<tr>
<td>Chapter 17</td>
<td>Probabilistic Evaluation of Counterfactual Queries</td>
<td>237</td>
</tr>
<tr>
<td>Alexander Balke and Judea Pearl</td>
<td>237</td>
<td></td>
</tr>
<tr>
<td>Abstract</td>
<td>237</td>
<td></td>
</tr>
<tr>
<td>17.1</td>
<td>Introduction</td>
<td>237</td>
</tr>
<tr>
<td>17.2</td>
<td>Notation</td>
<td>240</td>
</tr>
<tr>
<td>17.3</td>
<td>Party Example</td>
<td>241</td>
</tr>
<tr>
<td>17.4</td>
<td>Probabilistic vs. Functional Specification</td>
<td>242</td>
</tr>
<tr>
<td>17.5</td>
<td>Evaluating Counterfactual Queries</td>
<td>245</td>
</tr>
<tr>
<td>17.6</td>
<td>Party Again</td>
<td>248</td>
</tr>
<tr>
<td>17.7</td>
<td>Special Case: Linear-Normal Models</td>
<td>250</td>
</tr>
<tr>
<td>17.8</td>
<td>Conclusion</td>
<td>252</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>253</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 18 Causal Diagrams for Empirical Research (With Discussions) 255

Judea Pearl

Summary 255
Some key words 255
18.1 Introduction 255
18.2 Graphical Models and the Manipulative Account of Causation 258
18.3 Controlling Confounding Bias 262
18.4 A Calculus of Intervention 265
18.5 Graphical Tests of Identifiability 269
18.6 Discussion 275
Acknowledgments 277
18.A Appendix 278
References 279
18.I Discussion of ‘Causal Diagrams for Empirical Research’ by J. Pearl 282
18.II Discussion of ‘Causal Diagrams for Empirical Research’ by J. Pearl 283
18.IV Discussion of ‘Causal Diagrams for Empirical Research’ by J. Pearl 287
18.V Discussion of ‘Causal Diagrams for Empirical Research’ by J. Pearl 290
18.VI Discussion of ‘Causal Diagrams for Empirical Research’ by J. Pearl 292
18.VII Discussion of ‘Causal Diagrams for Empirical Research’ by J. Pearl 296
18.VIII Discussion of ‘Causal Diagrams for Empirical Research’ by J. Pearl 299
18.IX Discussion of ‘Causal Diagrams for Empirical Research’ by J. Pearl 300
18.X Rejoinder to Discussions of ‘Causal Diagrams for Empirical Research’ 303
Additional References 313

Chapter 19 Probabilities of Causation: Three Counterfactual Interpretations and Their Identification 317

Judea Pearl

Abstract 317
19.1 Introduction 318
19.2 Structural Model Semantics (A Review) 321
19.3 Necessary and Sufficient Causes: Conditions of Identification 331
19.4 Examples and Applications 342
19.5 Identification in Non-Monotonic Models 351
19.6 From Necessity and Sufficiency to “Actual Cause” 354
19.7 Conclusion 364
19.A Appendix: The Empirical Content of Counterfactuals 365
References 368
Chapter 20  Direct and Indirect Effects  373

Judea Pearl
Abstract  373
20.1 Introduction  373
20.2 Conceptual Analysis  375
20.3 Formal Analysis  380
20.4 Conclusions  390
Acknowledgments  390
References  391

PART V  CAUSALITY 2002–2020  393

Chapter 21  Introduction by Judea Pearl  395
References  396

Chapter 22  Comment: Understanding Simpson’s Paradox  399
Judea Pearl
22.1 The History  399
22.2 A Paradox Resolved  402
22.3 Armistead’s Critique  408
22.4 Conclusions  409
References  410

Chapter 23  Graphical Models for Recovering Probabilistic and Causal Queries from Missing Data  413
Karthika Mohan and Judea Pearl
Abstract  413
23.1 Introduction  413
23.2 Missingness Graph and Recoverability  414
23.3 Recovering Probabilistic Queries by Sequential Factorization  416
23.4 Recoverability in the Absence of an Admissible Sequence  418
23.5 Non-recoverability Criteria for Joint and Conditional Distributions  419
23.6 Recovering Causal Queries  420
23.7 Attrition  422
23.8 Related Work  423
23.9 Conclusion  424
Acknowledgments  424
References  424
23.A Appendix  426
### Chapter 24  Recovering from Selection Bias in Causal and Statistical Inference  433

*Elias Bareinboim, Jin Tian and Judea Pearl*

Abstract  433

24.1 Introduction  433
24.2 Recoverability without External Data  437
24.3 Recoverability with External Data  440
24.4 Recoverability of Causal Effects  444
24.5 Conclusions  447

Acknowledgments  447
References  447

### Chapter 25  External Validity: From Do-Calculus to Transportability Across Populations  451

*Judea Pearl and Elias Bareinboim*

Abstract  451

Key words and phrases  451

25.1 Introduction: Threats vs. Assumptions  452
25.2 Preliminaries: The Logical Foundations of Causal Inference  454
25.3 Inference Across Populations: Motivating Examples  461
25.4 Formalizing Transportability  465
25.5 Transportability of Causal Effects—A Graphical Criterion  471
25.6 Conclusions  475

25.A Appendix  477

Acknowledgments  478
References  478

### Chapter 26  Detecting Latent Heterogeneity  483

*Judea Pearl*

Abstract  483

Keywords  483

26.1 Introduction  483
26.2 Covariate-Induced Heterogeneity  485
26.3 Latent Heterogeneity between the Treated and Untreated  488
26.4 Three Ways of Detecting Heterogeneity  490
26.5 Example: Heterogeneity in Recruitment  495
26.6 Conclusions  497

Acknowledgments  498

Declaration of Conflicting Interests  498
Funding  498
Chapter 30  Causal Models and Cognitive Development  593

Alison Gopnik
References  601

Chapter 31  The Causal Foundations of Applied Probability and Statistics  605

Sander Greenland

Abstract  605
31.1 Introduction: Scientific Inference is a Branch of Causality Theory  606
31.2 Causality is Central Even for Purely Descriptive Goals  608
31.3 The Strength of Probabilistic Independence Demands Physical Independence  609
31.4 The Superconducting Super collider of Selection  610
31.5 Data and Algorithms are Causes of Reported Results  611
31.6 Getting Causality into Statistics by Putting Statistics into Causal Terms from the Start  612
31.7 Causation in 20th-century Statistics  613
31.8 Causal Analysis versus Traditional Statistical Analysis  614
31.9 Relating Causality to Traditional Statistical Philosophies and “Objective” Statistics  616
31.10 Discussion  618
31.11 Conclusion  619
31.A Appendix  619
Acknowledgments  620
References  620

Chapter 32  Pearl on Actual Causation  625

Christopher Hitchcock

Abstract  625
32.1 Introduction  625
32.2 Actual Causation  625
32.3 Causal Models and But-for Causation  626
32.4 Pre-emption and Lewis  631
32.5 Intransitivity and Overdetermination  634
32.6 Pearl's Definitions of Actual Causation  637
32.7 Pearl's Achievement  642
References  643

Chapter 33  Causal Diagram and Social Science Research  647

Kosuke Imai

33.1 Graphical Causal Models and Social Science Research  647
Chapter 34  Causal Graphs for Missing Data: A Gentle Introduction  655
Karthika Mohan
34.1 Introduction  655
34.2 Missingness Graphs  656
34.3 Recoverability  658
34.4 Testability  664
References  666

Chapter 35  A Note of Appreciation  667
Azaria Paz
35.1 A Magic Square  668
35.2 A Magic Shield of David  668

Chapter 36  Causal Models for Dynamical Systems  671
Jonas Peters, Stefan Bauer and Niklas Pfister
Abstract  671
36.1 Introduction  671
36.2 Chemical Reaction Networks and ODEs  675
36.3 Causal Kinetic Models  677
36.4 Challenges in Causal Inference for ODE-based Systems  681
36.5 From Invariance to Causality and Generalizability  682
36.6 Conclusions  683
Acknowledgments  684
References  684

Chapter 37  Probabilistic Programming Languages: Independent Choices and Deterministic Systems  691
David Poole and Frank Wood
37.1 Probabilistic Models and Deterministic Systems  693
37.2 Possible Worlds Semantics  694
37.3 Inference  700
37.4 Learning  703
37.5 Other Issues  704
37.6 Causal Models  705
37.7 Some Pivotal References  705
Chapter 41  Multivariate Counterfactual Systems and Causal Graphical Models  813

Ilya Shpitser, Thomas S. Richardson and James M. Robins

41.1 Introduction  813
41.2 Graphs, Non-parametric Structural Equation Models, and the g-/do Operator  820
41.3 The Potential Outcomes Calculus and Identification  833
41.4 Identification in Hidden Variable Causal Models  835
41.5 Conclusion  844
Acknowledgments  845
41.A Appendix  845
References  848

Chapter 42  Causal Bayes Nets as Psychological Theory  853

Steven A. Sloman

Abstract  853
42.1 The Human Conception of Causality  854
42.2 Core Properties  856
42.3 The Broader Perspective: The Community of Knowledge  859
42.4 Collective Causal Models  861
42.5 Conclusion  863
Acknowledgments  864
References  864

Chapter 43  Causation: Objective or Subjective?  867

Wolfgang Spohn

Abstract  867
43.1 Causation: A Bunch of Attitudes  867
43.2 The Model Relativity of Causation  871
43.3 Laws  874
43.4 Probability  878
Acknowledgments  886
References  886

Editors’ Biographies  889

Index  893