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A short note on the virtues of graphical tools

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An article by Fritz, Kenny, and MacKinnon (2014) analyzes the bias introduced in mediation problems when one ignores both measurement error and confounding. This note shows how their results can be obtained in a single step using the graphical tools introduced in Chen and Pearl (2014).

Computing the mediation bias

Fritz et al.'s model is shown in Fig. 1, with M_T denoting the true but unobserved mediator, M an observed proxy of M_T, and C denoting a confounder.

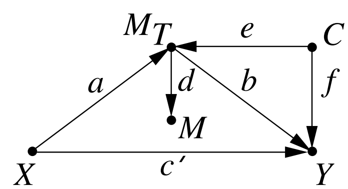

Figure 1:

The estimated mediated effect is given by the estimated total effect, β_{YX}, minus the estimated direct effect which, ignoring measurement error and conforming is given by the partial regression slope (assuming standardized variables throughout)

$$\beta_{YX\cdot M} = \frac{\beta_{YX} - \beta_{YM}\beta_{MX}}{1 - \beta^2_{XM}}$$

The backdoor condition for the graph in Fig. 1 dictates that $\beta_{YX} = c' + ab$ is an unbiased estimate of the total effect of X on Y (Chen and Pearl, 2014). Further, the graphical reading of β_{YX} and $\beta_{YX\cdot M}$ gives:

$$\beta_{YM} = d(b + ef + ac'), \qquad \beta_{YX\cdot M} = \frac{c' + ab - d(b + ef + ac')ad}{(1 - a^2d^2)}$$

Consequently, the Mediation Bias, defined as the difference between the estimated and true mediated effects (ab) becomes

$$\text{Mediation Bias} = (\beta_{YX} - \beta_{YX\cdot M}) - ab \tag{1}$$

$$= \frac{-ab(1-d^2) + ad^2ef}{1 - a^2d^2} \tag{2}$$

Clearly, zero bias is obtained when $b(1-d^2) = ad^2ef$ which coincides with Fritz et al.'s equation (15).

Discussion

Fritz et al. (2014) found it surprising that it is possible for different types of bias to virtually offset each other and in essence "two wrongs make a right" This is in fact a very common phenomenon in linear systems. It occurs, for example, when sample selection bias cancels confounding bias (Pearl, 2013, Eq. 20) or when two confounding paths have equal but opposite strength.

Fritz et al. compare this cancellation to the M-bias Pearl (2009, p. 186) which they interpret as "two confounders where correcting for one makes things worse than not correcting for either." One should remark that the M-bias depicted in Pearl (2009, p. 186) is a different phenomenon altogether. It represents a *barren proxy*, namely, a variable that has no influence on X or Y but is a proxy for factors that do have such influence. The bias introduced by conditioning on a barren proxy differs fundamentally from the bias introduced by disturbing the balance between two canceling misspecifications. The former is structural (persisting for every assignment of functions to the graph) while the latter is parametric – it depends on a delicate balance between the parameters in the model.

Finally, it is worth emphasizing again that the graphical tools introduced in (Chen and Pearl, 2014) are indispensible in analyzing, interpreting and communicating causal concepts such as "bias," "confounding," "mediation," and "structure."

References

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